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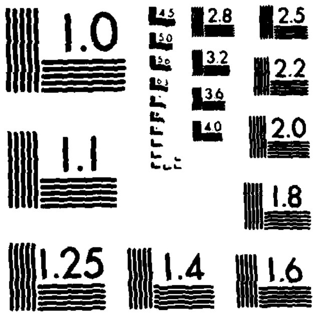
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## ABSTRACT

Cauchy problems for equations modelling non-Newtonian fluids are discussed and recent existence theorems for classical solutions, based on semigroup methods, are presented. Such existence results depend in a crucial manner on the symbol of the leading differential operator. Both "parabolic" and "hyperbolic" cases are discussed. In general, however, the leading differential operator may be of non-integral order, arising from convolution with a singular kernel. This has interesting implications concerning the propagation of singularities. In particular, there are cases where $C^\infty$-smoothing coexists with finite wave speeds.

AMS (MOS) Subject Classifications: 35A07, 35S10, 45K05, 76A10

Key Words: Viscoelastic Fluids, Type of Differential Operator, Wave Propagation, Cauchy Problems

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## SIGNIFICANCE AND EXPLANATION

This paper reviews recent existence results for initial value problems for equations modeling viscoelastic fluids. Motivated by the linear case, the equations describing these fluids are classified as parabolic, hyperbolic or intermediate.

The behaviour of these different types in linear wave propagation is discussed. In the hyperbolic case, propagation of shocks occurs, in the parabolic case the wave speed is infinite and singularities are smoothed out. The intermediate cases allow interesting situations where the wave speed is finite, but smoothing of singularities does occur.

Mathematical techniques applicable to the solution of initial value are discussed for both the parabolic and the hyperbolic case. In both cases, we discuss three-dimensional problems including treatment of the incompressibility condition. The intermediate case remain an interesting open problem.



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INITIAL VALUE PROBLEMS FOR VISCOELASTIC LIQUIDS

M. Renardy

## 1. Introduction

Viscoelastic liquids are characterized by constitutive laws allowing the stress to depend on the history of the deformation [3], [9], [17], [18]. This dependence is usually assumed to be local, i.e. the stress at the location of a given fluid particle depends only on the history of the deformation gradient at this same particle. Following Noll [17], such fluids are called "simple". The constitutive law is further restricted by the assumption that rigid body motions do not contribute to the stress (called the principle of frame-indifference) and by material symmetries. In this paper, I shall deal only with isotropic, incompressible materials.

For a mathematical existence theory, more needs to be known about the nature of the constitutive law. However, the precise form is not known for any particular material, and we must resort to models. Rheological models have been motivated by one or a combination of the following considerations:

1) Modification of linear theories to make them comply with frame-indifference.
2) Formal analogy with finite elasticity.
3) Kinetic theories of chain molecules.

From a mathematical point of view, the type of the equation, or, in other words, the nature of the leading differential operator is of particular interest. The rheological models can essentially be classified into three categories:

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hyperbolic case. The equations are transformed in such a way that they fit into the framework of known theorems due to Sobolevskii [28] and Kato [11-13], respectively. In both cases we deal with three-dimensional problems, including treatment of the incompressibility condition.

## 2. A problem in linear viscoelasticity

If the stress is a linear functional of the strain history, we can formally write the constitutive law in the form

$$\tau(t) = \int_{-\infty}^{t} \hat{a}(t-s)\gamma(t,s)ds \quad . \tag{2.1}$$

Here $\tau$ is the stress tensor and $\gamma$ is the gradient of the relative displacement (from the position at time $s$ to the position at time $t$). In particular, contributions resulting from different times $s$ superpose in an additive fashion. Boltzmann [3] suggested that the following restrictions should hold:

1) If the relative strain is always positive, so is the stress.
2) The strain from a more remote time always has a lesser influence than that from a more recent time.

This means that $\hat{a} > 0$, $\hat{a}' < 0$ in the sense of distributions. In this case, it can be shown [25] that

$$\hat{a}(\tau) = -\mu\delta'(\tau) + a(\tau) \quad , \tag{2.2}$$

where $\mu > 0$, and $a$ is a non-negative, non-decreasing function of $\tau > 0$.

This suggests that we classify constitutive laws according to the degree of the singularity of $\hat{a}$ at $\tau = 0$. The strongest possible singularity is the $\delta'$-distribution. If this term is absent, it is important whether $a$ is finite or infinite at $0$.

We shall consider the following particular problem: A fluid filling the half-space above an infinite plate is at rest for $t < 0$. At $t = 0$, the plat is suddenly set into uniform motion. We want to determine the motion for $t > 0$.

For a linear viscoelastic fluid, this leads to the equation

$$
\begin{aligned}
u_{tt}(x,t) &= \mu u_{xxt}(x,t) + \int_{-\infty}^{t} a(t-s)(u_{xx}(x,t) \\
&\qquad - u_{xx}(x,s))ds, \quad x > 0, \ t > 0 \ , \\
u(x,t) &= 0 \ , \quad t < 0 \ , \\
u(0,t) &= 1 \ , \quad t > 0 \ .
\end{aligned}
\tag{2.3}
$$

In addition to Boltzmann's restrictions, we assume that $a$ decays fast enough at $\infty$ so that the integral converges (in all rheological models I know of it decays exponentially).

If $\mu \neq 0$, the term $\mu u_{xxt}$ is the highest order term on the right hand side, and we would classify the equation as "parabolic". If $\mu = 0$ and $a$ is continuous, then the highest order term is $u_{xx} \cdot \int_0^\infty a(\tau)d\tau$, since the convolution is a differential operator of order $-1$. We would therefore call the equation "hyperbolic". Intermediate types arise from singular integral kernels, which act like fractional derivatives.

It is interesting to consider how these different types propagate singularities. Coleman and Gurtin [4] have shown (for $\mu = 0$) that, if there is a propagating shock front, its wave speed is $\sqrt{A}$ and its amplitude is $e^{-a(0)t/2A}$. Here $A = \int_0^\infty a(\tau)d\tau$.

It was shown in [16], [24] that indeed the singularity propagates as a shock if $\mu = 0$ and $a$ has $1 + \varepsilon$ derivatives in $L^1$. For $\mu \neq 0$, the shock is smoothed out and analytic solutions are obtained.

A class of kernels which act like fractional derivatives is given by

$$
a(\tau) = \sum_{n=1}^{\infty} e^{-n^\alpha \tau}, \quad \alpha > \frac{1}{2} \ . \tag{2.4}
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hyperbolic case. The equations are transformed in such a way that they fit into the framework of known theorems due to Sobolevskii [28] and Kato [11-13], respectively. In both cases we deal with three-dimensional problems, including treatment of the incompressibility condition.

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A class of kernels which act like fractional derivatives is given by

$$a(\tau) = \sum_{n=1}^{\infty} e^{-n^{\alpha}\tau}, \quad \alpha > \frac{1}{2} \quad . \tag{2.4}$$

Kernels of this nature are suggested by some molecular theories [7], [26], [31]. As $\tau \to 0$, a behaves like $\tau^{-1/\alpha}$. It is thus integrable for $\alpha > 1$ and not integrable for $\alpha < 1$. Thus the result of Coleman and Gurtin suggests an infinite wave speed for $\alpha < 1$, while for $\alpha > 1$ you expect a wave with finite speed but with no amplitude. This is in fact the case: If $\alpha > 1$, the solution $u$ is zero for $x > \sqrt{A}\,t$ and is analytic and not identically zero for $x < \sqrt{A}\,t$. Across the line $x = \sqrt{A}\,t$, however, there is no singularity and the solution is $C^\infty$ [24]. If $\alpha < 1$, the solution is analytic except at $t = 0$.

From the point of view of classification, we see that the hyperbolic term is still the leading order for $\alpha > 1$, however, the order of the correction is lower only by a fraction, not by one. If $\alpha < 1$, the right hand side of (2.3) is of fractional order.

## 3. An example of a parabolic model

The simplest class of "parabolic" models arises when a term of lower differential order is added to a Newtonian term. This is a common way of modeling dilute polymer solutions. Here you assume that the stress consists of a Newtonian part arising from the solvent, and some additional term coming from the dissolved polymer.

Before formulating the equations, we have to introduce some notation. Since the "simple fluid" is essentially a Lagrangian concept, it is natural to use Lagrangian coordinates. We denote those by $\zeta = (\zeta^1, \zeta^2, \zeta^3)$. $\underset{\sim}{\zeta}$ varies over a bounded domain $\Omega$ with a $C^4$-boundary. By $\underset{\sim}{y}(\underset{\sim}{\zeta}, t)$ we denote the position of the particle $\underset{\sim}{\zeta}$ at time $t$. The deformation gradient $F$ has components $F^i_j = \frac{\partial y^i}{\partial \zeta^j}$. It has to satisfy the incompressibility condition

$$\det F = 1 \quad . \tag{3.1}$$

The Cauchy strain tensor is defined by $\gamma = F^T F$. We write the constitutive law in terms of the upper convected stress $\pi$, which is related to the Cauchy stress $T$ by $T = F\pi F^T$. We assume that the constitutive law has the form

$$\pi = -p\gamma^{-1} - \eta \frac{\partial}{\partial t} (\gamma^{-1}) + \mathcal{F}(\hat{\gamma}) \quad . \tag{3.2}$$

$\mathcal{F}$ is a tensor-valued functional of $\hat{\gamma}$, the history of $\gamma$:

$$\hat{\gamma}(\underline{\zeta},t)(s) = \gamma(\underline{\zeta},t+s), \ s \in (-\infty,0] \quad . \tag{3.3}$$

If $\mathcal{F} = 0$, [3.2] describes a Newtonian fluid.

The equation of motion reads

$$\rho \ddot{y}^i = \frac{\partial}{\partial \zeta^s} \left( \frac{\partial y^i}{\partial \zeta^r} \pi^{rs} \right) + g^i(\underline{y},t) \quad , \tag{3.4}$$

and we impose Dirichlet conditions on the boundary

$$y^i(\underline{\zeta},t) = \phi^i(\underline{\zeta},t), \ \underline{\zeta} \in \partial\Omega \quad . \tag{3.5}$$

(The traction problem has also been considered [21]). We want to show that, if $\mathcal{F}$, $g$, $\phi$ and the initial history $\hat{y}(0)$ are "nice", then the initial value problem is uniquely solvable.

For the analysis, we introduce the following function spaces: $W^{p,k}(\Omega)$ denotes the usual Sobolev spaces, and we write $\underline{W}^{p,k}$, $\underline{\underline{W}}^{p,k}$ to indicate spaces of vector-valued or symmetric tensor-valued func-tions. $p$ is assumed to lie between 3 and 6. For the history dependence we use $C_b^{\lim}$, the space of bounded continuous functions which have a limit at $-\infty$. We write $\underline{C}_b^{\lim}$, $\underline{\underline{C}}_b^{\lim}$ for vector- and tensor-valued functions and $C_b^{\lim}(X)$ for functions taking values in the Banach space $X$.

The following describes the main ideas used in dealing with (3.1)-(3.5). For details, the reader is referred to [21].

1) Of course the basic idea is to treat (3.4) as a perturbation of the Navier-Stokes equation. The theory of the Navier-Stokes equation has been developed in the Eulerian frame. In order to carry over Eulerian

results to Lagrangian coordinates, we need sufficient smoothness of the transformation. This transformation, however, is itself one of the unknowns, so its smoothness has to be inferred from the equation itself. In order to satisfy this consistency requirement, it is advantageous to differentiate (3.4) with respect to time. We also differentiate (3.1) and (3.5) twice with respect to time. This leads to a system of equations for $y$, $u = \dot{y}$, $a = \ddot{y}$, $p$ and $q = \dot{p}$.

2) Equation (3.4) and the first time derivative of (3.1) can be used to express $u$ and $p$ in terms of $a$ and $\hat{y}$. This involves solving an elliptic system [29]. There is a gain of regularity: $u$ has two more derivatives than $a$.

3) The boundary and incompressibility constraints for $a$ can be reduced to a homogeneous form by determining an appropriate reference function and subtracting it from $a$. The determination of this reference function again involves solving an elliptic system.

4) A projection operator is used to eliminate $q$. This leaves an evolution problem for $y$, $b$ (the reference function from step 3) and $d$ (a new variable which replaces $a$).

5) In order to deal with the history dependence, we now regard the system from step 4 as an evolution problem on a history space. In doing this, we follow the following recipe. Suppose you have an equation of the form

$$\dot{z} = F(\hat{z},t) \quad , \tag{3.6}$$

where $\hat{z}(t)(s) = z(t+s)$, $s \in (-\infty,0]$. Then we define an operator $\hat{F}$ which maps $\hat{z}$ to the history of $F$: $\hat{F}(\hat{z},t)(s) = F(T_s\hat{z},t+s)$, where $T_s\hat{z}(t)(r) = \hat{z}(t)(s+r) = z(t+s+r)$. If the initial history satisfies the equation, (3.6) can be written in the form

$$\dot{z} = \hat{F}(\hat{z},t) \quad . \tag{3.7}$$

We can always make the initial history satisfy the equation by adding an appropriate term to the body force.

6) This finally leads to an evolution problem for $\hat{y}$, $\hat{b}$, $\hat{d}$. When posed on the space $C_b^{\lim}(\underline{W}^{p,4}(\Omega) \times \underline{W}^{p,2}(\Omega) \times \mathring{\underline{L}}^p(\Omega))$ where $\mathring{\underline{L}}^p$ denotes the subspace of divergence-free vector fields with zero normal component on the boundary, this problem satisfies the assumptions of a theorem due to Sobolevskii [28] on abstract quasilinear parabolic equations. The essential point in proving this is of course the fact that the Stokes operator generates an analytic semigroup in $\underline{L}^p$ ([8], [30]).

## 4. An example of a hyperbolic model

The K-BKZ model [2], [14] is motivated by an analogy with finite elasticity. The constitutive law for an incompressible elastic material has the form

$$\pi = -p\gamma^{-1} + \frac{\partial W}{\partial I_1}\gamma_0^{-1} - \frac{\partial W}{\partial I_2}\gamma^{-1}\gamma_0\gamma^{-1} \quad , \tag{4.1}$$

where $\gamma_0$ is a constant tensor and $W$ is a scalar function of $I_1 = \operatorname{tr}(\gamma\gamma_0^{-1})$ and $I_2 = \operatorname{tr}(\gamma^{-1}\gamma_0)$. Kaye [14] and Bernstein, Kearsley and Zapas substituted the following for a viscoelastic material

$$\pi = -p\gamma^{-1} + \int_{-\infty}^{t} a(t-\tau)\Big[\frac{\partial W}{\partial I_1}\gamma^{-1}(\tau) - \frac{\partial W}{\partial I_2}\gamma^{-1}(t)\cdot \gamma(\tau)\gamma^{-1}(t)\Big]d\tau \tag{4.2}$$

with $I_1 = \operatorname{tr}(\gamma(t)\gamma^{-1}(\tau))$ and $I_2 = \operatorname{tr}(\gamma^{-1}(t)\gamma(\tau))$. The model thus assumes that every previous state of the material is like a temporary equilibrium state to which the material likes to revert; the influences of all the previous are assumed to superpose in an additive fashion.

We assume that the kernel $a$ is positive and smooth, including $t-\tau = 0$. Let $F$ denote the relative deformation gradient, $F^i_p = \frac{\partial y^i(t)}{\partial y^p(\tau)}$ and let $\bar{F}^p_i$ denote the entries of $F^{-1}$: $\bar{F}^p_i = \frac{\partial y^p(t)}{\partial y^i(\tau)}$. The equation of motion can be written in the form

$$\rho \ddot{y}^i = -\frac{\partial p}{\partial \zeta^s}\frac{\partial \zeta^s}{\partial y^i} + \frac{1}{2}\int_{-\infty}^{t} a(t-\tau)$$

$$\cdot \frac{\partial^2 W}{\partial F^i_p \partial F^j_r}\Big[\frac{\partial^2 y^i}{\partial \zeta^q \partial \zeta^s}\frac{\partial \zeta^q}{\partial y^r(\tau)}\frac{\partial \zeta^s}{\partial y^p(\tau)} + \frac{\partial y^j}{\partial \zeta^q}$$

$$\frac{\partial}{\partial \zeta^s}\Big(\frac{\partial \zeta^q}{\partial y^r(\tau)}\Big)\frac{\partial \zeta^s}{\partial y^p(\tau)}\Big]\, d\tau + g^i \quad , \tag{4.3}$$

$$\det\Big(\frac{\partial y^i}{\partial \zeta^j}\Big) = 1 \quad .$$

The "hyperbolic" character of (4.3) is guaranteed by a strong ellipticity condition, which has the same form as in elasticity

$$\Big(\frac{\partial^2 W}{\partial F^i_p \partial F^j_r} + K\bar{F}^p_i\bar{F}^r_j\Big)\lambda^i\lambda^j\mu_p\mu_r$$

$$\geq c|\lambda|^2\,|\mu|^2 \quad , \quad c > 0 \quad . \tag{4.4}$$

for large enough $K$. This condition is expressed in terms of $F$ and has a rather indirect form in terms of $I_1$ and $I_2$. However, it is possible to give the following sufficient condition: (4.4) holds if $W$ is monotone in both $I_1$ and $I_2$, strictly monotone in at least one of them, and $W$ is a convex function of $\sqrt{I_1}$ and $\sqrt{I_2}$.

In [23], I proved a local existence theorem for (4.3) posed in all of space, assuming that (4.4) holds. The analysis proceeds in $L^2$-type spaces, i.e. we deal with solutions for which $\underline{y} - \underline{\zeta} \to 0$ at infinity.

Equation (4.3) can formally be written as a non-delay evolution problem on a history space. In this case, we have chosen a way of doing this which is different from the one adopted in chapter 3. For each $t > 0$, we put $\hat{\underline{y}}(t)(\sigma) = \underline{y}(\sigma t)$. That is, $\hat{\underline{y}}$ only contains the history from the initial time $t = 0$ up to the present time, not the history for $t < 0$ (which is of course considered known).

Again, it is advantageous to differentiate the equation with respect to time, in this case we do this twice. Lower order time derivatives can be expressed in terms of higher order time derivatives by solving systems which are elliptic in the sense of Agmon, Douglis and Nirenberg [1]. For instance, (4.3) can be regarded as a nonlinear elliptic equation for $y$ and $p$, if you presume $\ddot{y}$ is known. One then looks at the second time derivative of (4.3). The variable $\ddot{p}$ can be eliminated from this equation using the Hodge projection. One finally ends up with an evolution problem for the two variables $\ddot{\hat{\underline{y}}} - \frac{\lambda}{\rho}(\hat{\underline{y}} - \underline{\zeta})$ and $\dddot{\hat{\underline{y}}} - \frac{\lambda}{\rho}\dot{\hat{\underline{y}}}$, where $\lambda$ is an appropriately chosen constant.

In this system the leading operator is elliptic and generates a quasi-contraction semigroup. An existence theory for systems of this kind is provided by Hughes, Kato and Marsden [11]. They study quasilinear evolution equations in a reflexive Banach space $X$ of the form

$$\dot{u} = A(t,u)u + f(t,u) \quad , \tag{4.5}$$

where $A(t,u)$ is a possibly unbounded linear operator and $f$ is a bounded nonlinear term. The essential assumptions of the theory are that $A$ generates a quasi-contraction semigroup (in a norm which is allowed to depend on $t$ and $u$), and that there is an "elliptic" operator $S(t,u)$ such that $SAS^{-1} - A$ is bounded. Here "elliptic" means that $S$ is a bijection from an embedded space $Y$, which is contained in the domain of $A$ and independent

of $t$ and $u$, onto $X$. In the present case, $A$ turns out to be elliptic, and we can simply take $S = A$. The theorem of Hughes, Kato and Marsden guarantees the local existence of solutions to the initial value problem, if we assume sufficient smoothness of the data. These solutions can be obtained by the iteration

$$\dot{u}^{n+1} = A(t,u^n)u^{n+1} + f(t,u^n) \quad . \tag{4.6}$$

The most interesting question from the rheologist's point of view is probably whether (4.4) is valid. While it is clear that (4.4) would hold for small deformations in any reasonable model, the global validity is not so clear. *Some popular rheological* models, e.g. $W = I_1$ or $I_2$, always satisfy (4.4). For other models, however, (4.4) fails at large deformations. This is not necessarily bad. If (4.4) fails, the evolutionary character of the equations can be lost (cf. also [27]), and one expects something strange to happen to the material. In fact, strange things do happen at high shear rates, and loss of hyperbolicity may be a possible explanation for these phenomena, generally known as "melt fracture".

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Cauchy problems for equations modelling non-Newtonian fluids are discussed and recent existence theorems for classical solutions, based on semigroup methods, are presented. Such existence results depend in a crucial manner on the symbol of the leading differential operator. Both "parabolic" and "hyperbolic" cases are discussed. In general, however, the leading differential operator may be of non-integral order, arising from convolution with a singular kernel. This has interesting implications concerning the propagation of singularities. In particular, there are cases where $C^{\infty}$-smoothing coexists with finite wave speeds.

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